THE PRINCIPLES

OF THE CELESTIAL

MECHANISM OF THE SOLAR SYSTEM

EPITOMIZED;

WHEREIN ARE CONTAINED

RULES FOR DETERMINING THE CENTRIPETAL FORCES OF

THE PLANETS AND OF THEIR SATELLITES,

AND WHEREBY THE LENGTH OF THE EARTH'S CIRCUMFERENCE,

AND OF A DEGREE OF A GREAT CIRCLE ON ITS SURFACE,

MAY BE MORE EXACTLY DETERMINED THAN BY FORMER METHODS;

WITH

EXAMPLES AND CALCULATIONS.

BY THE AUTHOR OF

"COMMENTARIES ON THE PRINCIPIA OF SIR ISAAC NEWTON," &c.

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MECHANISM

OF

THE SOLAR SYSTEM.

CHAPTER I.

In the rude and uncivilized ages, before the dawn of science, the earth was considered as a body of an infinite extent on its surface, then deemed a plane superficies, except so far as the level was interrupted by mountains, valleys, and other observable deviations. Mankind then thought that the earth was of infinite extent in depth below its surface. Being only very partially acquainted with the old world, consisting of Europe, Asia, and Africa, and ignorant of the existence of America, the new world, and finding, as far as their

observations could extend, that all the parts of the continent of the old world with which they were acquainted were limited, and that its coasts were environed by the ocean, they would consider this expanse of ocean either interminable, or as admitting, at some distance, of some new and unknown world, either of an infinite extent on its surface, or again bounded onwards by another ocean; and so in infinitum, land and sea, and land and sea, unless either land or sea was ultimately infinite. As far as they could judge from the evidence of their senses, the earth was at rest, because they were insensible to any motion of it: they found its surface wherever they went, a plane superficies with undulations: they found this surface unbounded, taking land and sea together, and they thought it infinite, but only as a plane surface; they had no reason to believe or suspect that the earth was a globe or sphere.

In the pastoral ages which succeeded, the shepherds and herdsmen who watched

their flocks and herds at night, were diligent observers of the stars. They observed that the whole celestial hemisphere turned round once in twenty-four hours; this they called the diurnal motion of the heavens. They also observed that the sun had, besides his diurnal motion, another motion, by which he moved in an undeviating circle, called the Ecliptic, round the celestial hemisphere, completing the revolution in one year; this was called the sun's annual motion.

Five stars in the celestial hemisphere were observed also to possess, besides the diurnal motion, another motion, by which they completed their revolution round the sun in different periodic times. These stars, called planets, afterwards received the names of Mercury, Venus, Mars, Jupiter, and Saturn; Mercury being the nearest to the sun, Venus next, and so on. It was observed, also, that Mercury and Venus were sometimes between the sun and the earth, which was not the case with any one of the three other planets.

The eclipses of the sun and moon occurring annually, showed that the moon is a satellite revolving round the earth at a distance much less than that of the earth from the sun.

From these and other phenomena they concluded that the earth is also a planet; that it is of a spherical form, and that its orbit is between those of Venus and Mars; Mercury and Venus were hence denominated *inferior* planets, and Mars, Jupiter, and Saturn *superior* planets, in the system. By observation, the earth's periodic time of revolution was found to be three hundred and sixty-five days, six hours, nearly, and that of the moon round the earth to be somewhat less than twenty-eight days. The planes of the orbits of the planets were found to be not quite coincident, but to deviate very little one from another.

The ancients were altogether unacquainted with the nature, and even the existence, of the forces which occasion the motion of these heavenly bodies.

The other stars which had no apparent

motion but the diurnal one, were considered *fixed* stars, as their relative position to each other does not vary.

The ancient Chaldæans divided the ecliptic or great circle of the heavens in which the earth revolves round the sun into twelve equal parts, by the following simple method. They filled a large vessel with water, and when the first star in the constellation Aries, in the ecliptic, was on the meridian, they let the water begin to run through a small hole in the bottom of the vessel, into another large vessel, and continue running until the same star came to the meridian again, which was exactly in twenty-four hours, when they stopped the hole. They then divided the quantity of water in the second vessel into twelve equal parts, and having emptied the first vessel of whatever surplus water it might still contain, they poured into it one of the twelve parts, and when the first star in Aries came to the meridian again, they opened the hole, and observed what star, in or near the ecliptic, was on the meridian at the moment when

the twelfth part of the water had run out. The distance in the heavens between the two stars was the first sign, Aries, of the Zodiac. In the same manner, they marked out the other eleven signs of the Zodiac, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, &c.

Such was the knowledge or belief of Pythagoras, and the ancients before his time; and the system, placing the sun in the centre of the orbits of the planets, and now called the Pythagorean system, seems to have been generally received until the time of Ptolemy, the Egyptian astronomer, who flourished about thirty years before the Christian era. Pythagoras flourished about six hundred years before Ptolemy's time.

Aristotle, about two hundred years after the death of Pythagoras, observed that all bodies at the surface of the earth possess two kinds of appetence or tendency (*appetentia*); one, by which they tend to the centre of the earth, and the other, by which they are driven from this centre. This opinion, which appears to be the earliest

notion of the centripetal and centrifugal forces, did not, in those early ages, lead to any analogy with the motions of the planets in general; an enquiry reserved for the genius of Newton.

The Pythagorean system was superseded about thirty years before the Christian era by the Ptolemaic system, which places the earth in the centre. As the Ptolemaic system is now known to be erroneous, it would be an useless labour to state the particulars of it, which would only be a detail of ingenious absurdities. However, it was destined to universal credence for nearly 1600 years, when an accident occasioned the restoration of the Pythagorean, under the name of the Copernican, system.

CHAPTER II.

OF THE REVIVAL AND FINAL ESTABLISHMENT OF THE PYTHAGOREAN SYSTEM.

Nicholas Copernicus, a native of Thorn, in Prussia, found it stated in Plutarch's life of Numa, that the old Pythagoreans in Numa's time considered the sun in the centre; and the earth and the other planets as revolving round the sun. Copernicus adopted this system, and wrote a treatise in support of it in the early part of the sixteenth century, which he published a short time before his death, A. D. 1543, after the Pythagorean system had been exploded nearly 1600 years. The revived system gained ground, and was adopted by most astronomers, in that and the succeeding age. Tycho de Brahe, however, a Danish astronomer, who died A. D. 1601,

still adhered to the Ptolemaic system, with some modifications. His system is called the Tychonic system, but it is now exploded, following the fate of the Ptolemaic, on which it was founded.

In the early ages, astronomers had exactly determined, by observation, the *periodic times* of the earth and the other planets, and had estimated their *mean distances* from the sun with considerable accuracy. John Kepler, a native of Wiel, in the Duchy of Wirtemburg, found (A. D. 1618), by comparing the periodic times of the planets with their mean distance from the sun, that the squares of the times are as the cubes of the mean distances; a very important discovery, which enabled modern astronomers to determine more accurately than before, the respective mean distances of the planets from the sun. Kepler adopted the Pythagorean system, after the example of Copernicus.

In the mean time, Galileo, the son of a nobleman of Pisa, discovered (by means of the telescope, invented by Jansen,

and which Galileo improved by one constructed by himself), discovered, I say, the satellites of Jupiter, the rings of Saturn, and other phenomena not previously observable. Galileo died A. D. 1642, being the year in which Newton was born.

Sir Isaac Newton was born at Woolstrop, in Lincolnshire. He was descended from the eldest branch of the family of Sir John Newton, Bart., who possessed the manor of Woolstrop about two centuries before the birth of Newton. The family was not originally of Lincolnshire, but of Newton-in-the-Willows, in the county of Lancaster, from which place they removed, first to Westley, in Lincolnshire, and afterwards to Woolstrop.

Newton, the bent of whose genius led him to the investigation of astronomical subjects, discovered, at the early age of about twenty-six, that the descent of bodies falling near the surface of the earth was the effect of a mutual attraction between the earth and the bodies falling. He was hence led to infer that there was a mutual attraction of

the same kind between the earth and the moon, and between the sun and the earth, and also between the sun and the other planets. This great discovery, which he denominated *gravitation*, was the extension and perfection of that principle of a centripetal force, which Aristotle had observed as existing at the earth's surface.

Gravitation, if it existed alone in the solar system, would, by its tendency, bring the sun and all the planets and their satellites into contact together, and the revolutions of the planets round the sun require the existence of some other equal force which, by a concurrent action with gravitation, would prevent that contact. This concurrent force as to each planet is equal to its force of gravitation; its direction is at right angles to that of gravitation, and in the plane of the planet's orbit: it is called the centrifugal force of the planet; the planet's gravitation being considered its centripetal force. Newton's discovery of the universal principle of gravitation, led

him also to the discovery of the centrifugal forces, which prevent the collision of the planets with the sun, and enable them to persevere in their revolutions round the sun.

Gravitation, or the centripetal force, is an affection existing between all material substances, and is continually acting. On the other hand, the centrifugal force is a constant and continuing force, which originated in an instantaneous projectile impulse communicated to each planet, at the moment when it at first became impressed with the influence of gravitation. Such was the principle discovered by Newton, of the celestial mechanism of the solar system.

Newton also discovered the analogy between the velocities of the planets, and their mean distances from the sun, which analogy had not been observed by Kepler; Newton's analogy being, that the velocities are inversely in the subduplicate ratio of the distances; or, $V : v :: \frac{1}{D^{\frac{1}{2}}} : \frac{1}{d^{\frac{1}{2}}}$, Kepler's analogy is thus expressed:

$$T^2 : t^2 :: D^3 : d^3;$$

from which formulæ we derive by elimination the following expressions:

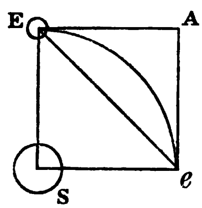
$$\mathrm{T}^{\frac{2}{3}} ; t^{\frac{2}{3}} : \mathrm{D} : d;$$
$$\mathrm{D}^{\frac{1}{2}} ; d^{\frac{1}{2}} : \mathrm{T}^{\frac{1}{3}} : t^{\frac{1}{3}}; \text{ and}$$
$$\mathrm{V} : v :: \frac{1}{\mathrm{T}^{\frac{1}{3}}} : \frac{1}{t^{\frac{1}{3}}}.$$

It is obvious that the circumferences of the orbits of the planets, or the spaces which they describe in their periodic times, are the resultants of the composition of their centripetal and centrifugal forces. Now the velocity is the measure of the sum of the two equal forces[1], and if F represents the sum, $\mathrm{F} = \mathrm{V}$. Hence $\mathrm{F} : f :: \mathrm{V} : v$; and because $\mathrm{V} : v :: \frac{1}{\mathrm{D}^{\frac{1}{2}}} : \frac{1}{d^{\frac{1}{2}}} : \mathrm{F} : f :: \frac{1}{\mathrm{D}^{\frac{1}{2}}} : \frac{1}{d^{\frac{1}{2}}}$; that is, the sum of the two forces are inversely in the subduplicate ratio of the mean distances from the sun. It is obvious that the ratios of the sums of the forces are the same as the ratios of the halves of those sums, that is, as the ratios of the centripetal, and also of the centrifugal, forces.

Having thus determined the ratios of

[1] See Whewell's Elementary Treatise on Mechanics, fifth edition, page 141.

the centripetal forces, it follows, that if we ascertain the centripetal force, or the force of gravitation of any one planet, we can, by the above formulæ, find the centripetal forces of all the other planets. For instance, let it be required to determine the force of the earth's gravitation towards the sun.



Let E represent the earth at any point in its orbit, S the sun, and the right line E S the earth's mean distance from the sun: the earth will describe one-fourth of its orbit, considered circular, in one-fourth of its periodic time by the concurrent action of the centripetal and centrifugal forces; that is, it will describe the arc E *e* in the time, at the end of which it will come to the point *e*, being at the same distance from S, as from E to S. Now the effect will be the same, if we consider the earth at the point E as impressed by two equal uniform forces, represented by the equal right lines E A, E S, acting at right angles to each other, one of which, E S, acting alone,

would impel the earth from E to S, and the other, E A, also acting alone, from E to A, in the time in which the earth arrives at the point *e;* that is, the arc E *e* may be resolved into the two equal uniform forces E A and E S, which are therefore equal together to the centripetal and centrifugal forces together, and E S, the distance of the earth from the sun, is equal to the centripetal force of the earth towards the sun, and is measured by one-fourth of its periodic time; that is, $F = \frac{D}{\frac{1}{4}T}$.

In like manner may the centripetal forces of all the other planets be determined by the aid of the last formula (which I call the *direct* method), or by the analogy $E : f :: \frac{1}{D^{\frac{1}{2}}} : \frac{1}{d^{\frac{1}{2}}}$, which is the inverse method; but great advantage will arise if both methods be adopted; for if the results correspond, it will follow that the distances of the planets from the sun are correctly estimated; but if the results differ, the difference arises from the distances

being either overrated or underrated, which results will suggest a medium converging point giving the distances more exactly; for if the distance is overrated of the planet whose centripetal force is sought, the direct method will give a greater centripetal force than the true one; and the inverse method will give a less than the true one; and conversely, if the distance is underrated.

The *mere* forces of gravitation, considered independently of their *action*, are inversely as the distances from the sun; for the attraction of gravitation between the sun and each planet emanates from the sun's hemisphere as a cone of which the vertex is the point where the planet's shadow ends, and diminishes in the ratio in which the altitude of the cone (which is nearly the same as the planet's distance) is increased, which is an inverse ratio; but the *action* of the forces is a direct ratio, which, according to what is above premised, is as the square roots of the distances. Hence the force and action combined of

gravitation on the planets may be expressed by the formula $F = \frac{D^{\frac{1}{2}}}{D}$; in which F represents the force and its action combined; for, dividing both the numerator and the denominator by $D^{\frac{1}{2}}$, $F = \frac{1}{D^{\frac{1}{2}}}$, and, consequently, $F : f :: \frac{1}{D^{\frac{1}{2}}} : \frac{1}{d^{\frac{1}{2}}}$, as we have before shown.

The following table contains the centripetal forces of the planets therein named, calculated by the direct method abovementioned.

TABLE.

Planets.	Mean distances from the Sun in miles.	Periodic times in our days.	Gravitating forces in miles per diem.
Mercury .	26,800,000	88	1,672,727
Venus . .	68,610,000	$224\frac{2}{3}$	1,221,592
The Earth	95,008,098	$365\frac{1}{4}$	1,040,411
Mars . . .	144,865,000	687	842,238
Jupiter . .	494,310,000	$4,332\frac{1}{2}$	456,375
Saturn . .	906,440,000	10,759	337,000
Uranus . .	1,825,000,000	$30,737\frac{2}{3}$	237,506

The author of this epitome has found by arithmetical computation, that the gravitating or centripetal forces of the planets,

collected in the above table, are to each other inversely as the square roots of their distances from the sun; with some inconsiderable exceptions, which exceptions seem to arise from the distance of Venus from the sun being somewhat underrated in the table, and the distance of Mars from the sun being therein somewhat overrated.

We now proceed to apply the doctrines we have as above endeavoured to establish to practical purposes.

PROPOSITION I.—Problem I.

To determine the centripetal force of any planet, whether primary or secondary, to its centre of gravity.

Rule I.

Divide the mean distance of the planet from the centre of its orbit by one-fourth of its periodic time, and the quotient will be the centripetal force that is sought.

For, as we have shown, $F = \frac{D}{\frac{1}{4}T}$.

This Rule is to be used when the mean distance and the periodic time are both given quantities; but when only the mean distance of any two planets, and the centripetal force of one of them are given, we must resort to

RULE II.

State the inverse proportionals as follows: as the square root of the less distance is to the square root of the greater distance, so is the force of the further planet to the force of the nearer one; and proceed as in the rule of three.

Example 1.

To determine the centripetal force of the moon towards the earth.

The moon's estimated distance from the earth's centre is 240,000 miles; her periodic time is 656 hours, nearly, one-fourth of which is 164 hours. Divide 240,000 by 164; $\frac{240,000}{164}=1453$ miles per horam nearly, which is the centripetal force of the moon towards the earth, by the direct method.

Example 2.

To determine the centripetal force of the earth at its surface.

The ratio of the distance of the moon from the centre of the earth, to the distance of the earth's surface from its centre, is as 240,000 to 4000, or as sixty to one. As the problem stated in this second example gives the periodic time of the moon, but not the periodic time of any body supposed to revolve round the centre of the earth, near its surface, we must solve this problem by the inverse method of Rule II., as follows:

$$1\tfrac{1}{2} : 60\tfrac{1}{2} :: 1463 : 1463 \times 774$$
$$:: 1463 : 11323 \text{ miles per horam.}$$

(*The Answer.*)

We may, however, solve this problem by determining, first, what would be the periodic time of a body revolving about the centre of the earth, near its surface, as follows:

The estimated mean distance of the surface of the earth from its centre is 4000

miles, and the periodic time of the body revolving near the earth's surface would be, according to Kepler's analogy, 1·41 hours nearly, one-fourth of which is 3525 hours. Divide the distance, 4000 miles, by ·3525 hours, $\frac{4000}{\cdot 3525}$ = 11,346 miles per horam, the centripetal force of the earth's gravitation at its surface. The trifling difference between the last two results is owing to my disregarding in the calculation the fractional part of an hour (43′) in the moon's periodic time.

PROPOSITION II.—Problem II.

To find the length of the circumference of a great circle of the sphere of the earth.

The earth's diameter = 4000 × 2 = 8000 miles, which multiplied by 3·1416 (being the ratio of the circumference to the diameter), gives the length of the circumference, = 25,132·8 miles; which is usually estimated at 24,912 miles.

Corollary.—The length of a degree of a meridian, or great circle, of the earth is $\frac{25132}{360}$, usually estimated at 698 miles.

From what has been premised, we may easily determine the constant ratio between the velocity and the centripetal force of each planet; which is the ratio of the arc of a quadrant of the circle to the radius; that is, in the case of the earth, as $\frac{25132 \cdot 8}{4} = 6283 \cdot$ to 4000, or, as 1·57 to 1 nearly.

In my work entitled "A New Analogy for determining the Distances of the Planets from the Sun," (Whittaker and Co., 1842,) I have shown that the analogy which subsists between the distances and velocities of the primary planets, also subsists between the distances and velocities of the satellites of Jupiter, and those of Saturn; but the force of the gravitation of the sun is thirty-five times that of Jupiter, and sixty-eight times that of Saturn, and 582 times that of the earth, very nearly.

CHAPTER III.

Sir Isaac Newton having, in the Principia, advanced the theory, that the centripetal forces of the planets are inversely in the duplicate ratio of their distances from the sun, which is a different theory from that advanced, and, as I hope, demonstrated in this epitome, I subjoin the following observations, as containing the principal grounds of objection to the supposed proofs of his theory as stated in the Principia.

1. It is a part of Newton's theory, that the centripetal and centrifugal forces of each planet must be equal, in order to retain the planet in its orbit; and he found that the centripetal force of the moon towards the earth, as computed according to his theory, was very insufficient for this purpose.

This consideration induced him to desist from pursuing his theory for twelve years, and when he resumed it, he left it liable to this formidable objection, which still remains. He says in Book III, Principia, Prop. 4, that he had calculated the centripetal force of the moon to be $15\frac{1}{12}$ feet per minute, which would be 905 *feet* per horam. But to balance the moon's centrifugal force would require a centripetal force of 1420 miles per horam.

2. Corol. iv. to Prop. 1, Principia, is contradictory to Prop. 4. For by Corol. iv. Prop. 1, $F : f :: V^2 : v^2$; but by Prop. 4,

$F : f :: \frac{V^2}{D} : \frac{v^2}{d}$; which is a different ratio: and what is very remarkable is, that the latter ratio purports to be demonstrated from the other.

3. Newton in Corol. iv. Prop. 1, assumes that the versed sines of arcs represent the centripetal forces. The author of this epitome was led hastily to adopt this opinion in his "Commentaries on the Principia," (Whittaker and Co., 1846,) but on further

consideration, he finds that the opinion is not founded either on fact or reasoning, and is in reality a false inference from an illusory appearance.

4. In Corol. iv. Prop. 1, Newton shows that the versed sines are as the squares of the arcs or velocities, when both quantities are evanescent; whereas in Prop. 4, Book III, Principia, he computed the centripetal force of the moon as represented by the versed sine of an arc described by the moon in one minute, which is greater than an evanescent arc, and therefore the ratio does not hold good.

5. In Prop. 4, Book III, Principia, Newton calculates the centripetal force of the moon to be $15\frac{1}{12}$ feet in one *minute*. According to his own theory, and taking (as he does) the descent of bodies at the earth's surface, viz. $15\frac{1}{2}$ feet, to be the measure of the earth's centripetal force at its surface, the force of the earth's gravitation on the moon would be $15\frac{1}{12}$ feet per horam.

6. In Prop. 4, Book III, Principia, Newton compares the descent of the moon consi-

dering her centrifugal force, abstracted and gravitating by the influence of her centripetal force only, with the descent of bodies falling near the earth's surface, which gravitate by the joint effect of the centripetal and centrifugal forces of the earth; which is a comparison of dissimilar cases. Bodies falling near the earth's surface gravitate, because the earth's centripetal force is greater there than its centrifugal force. The moon's centripetal and centrifugal forces are equal to each other, which equality of the forces retains her in her orbit.

THE END.

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